# The Logic of Confusion

Jean-Yves Béziau
CSLI - Stanford University
Stanford, CA, USA

**Abstract** *The logic of confusion is a way to handle together incompatible "viewpoints". These viewpoints can be information data, physical experiments, sets of opinions or believes. Logics of confusion are obtained by generalizing Jaskowski-type semantics and combining it with many-valued semantics.*

*Keywords:* paraconsistent logic, discussive logic, many-valued logic, quantum physics

## 1 Subjectivism, objectivism and confusion

When we have different incompatible viewpoints regarding something, we may consider that these viewpoints are in fact about different things (subjectivism) or that they are about the same thing (objectivism) but that there is only one which is right or true (classical objectivism), or that each one maybe true (paraconsistent objectivism).

In this paper we will present a general way to handle together incompatible viewpoints which promotes objectivism via paraconsistent logic. This general framework will be called *logic of confusion*. The reason for the name is that we want to put (*fusion*) together (*con*) different viewpoints. The word "confusion" in English has a pejorative connotation. To have at the same time incompatible viewpoints generally leads indeed to confusion, since it seems that there is no way to control the situation, because contradiction trivializes. Paraconsistent logic is a way to handle contradictions and therefore can be used to develop a positive theory of confusion.

This does not mean that we want to promote confusion. The logics we present here can be seen as a way to control it. On the other hand one can say that classical logic encourages confusion because it provides no way to deal with it.

The logic of confusion encompasses various different particular logics. One of them is the *discussive logic* of Jaśkowski. The basic motivation of this logic is to handle discussion groups of people with incompatible opinions.

Some logics of confusion that we will present can be applied to *quantum physics*. According to quantum physics some objects may have contradictory properties (cf. the wave/particle duality), although since they do not manifest in the same experiment, we cannot say that we have a blatant contradiction.

To explain this phenomenon, Niels Bohr introduced the so-called "principle of complementarity". In fact there is no "principle of complementarity". "Complementarity" is just a word for some philosophical ideas, for an insight that Bohr was never able to formulate in a clear way (see e.g. [2]). For Bohr, complementarity did not restrict to modern physics, but also applied to biology or psychology and was in fact a kind of general philosophical perspective. By contrast many working physicists endorse the so-called *Copenhague Interpretation* in order to get rid of philosophical speculation. This leads them to a kind of antirealist postion. However the idea of people like Heisenberg and Weizsäcker was rather that a new logical language was necessary to describe the reality of quantum objects (see e.g. [17], Chapter X).

In the logic of confusion, the different experiments are simply considered as different viewpoints that can coexist without trivialization. The logic of confusion can be seen as the logic of complementarity (On the relation between paraconsistency and complementarity, see also [15], and for more discussions about the connection between logic and physics, see [6]). But the logic of confusion fits well also with the ideas of David Bohm. It is well-known that Bohm presented a rival physical theory whose main purpose was to support realism. However Bohm's realism was not classical, for example he wrote:

> "When we come to consider the 'totality of all that is', however, our primary concern is, as we have seen, not with conditionned things but with the unconditionned reality that is the ultimate ground of all. Here, the rules enunciated by Aristotle break down, in the sense that there is not even a limited domain or set of conditions under which they could apply".([11], p.60)

The logic of confusion can be seen also as a logic of information. In artificial intelligence one has to deal with packages of contradictory information and also, most of the time, sets of incomplete information. Here we present a logic of confusion which is both paraconsistent and paracomplete and that can be therefore applied to such common situation. We built this logic by combining the Jaskowski-type semantics underlying the logic of confusion with many-valued semantics.

Some relations between these two types of semantics have also been investigated by W.Carnielli (see e.g. [13]), but in his approach many-valued semantics appear at the external level, as another way to describe Jaskowski-type semantics; here many-valued semantics appear rather at the internal level and the two types of semantics are mixed. The analysis of Carnielli can also be applied to many-valued logic of confusion, in this case many-valuedness will appear at the two levels.

Another approach which has some connections with the one presented here and which is based on a similar intuition is the one of A.Buchsbaum and T.Pequeno (see e.g. [12]).

## 2 Logic of confusion : general definition and general results

We consider a set of functions $\Omega$ from the set of formulas $\mathcal{F}$ into $\{\bot, \top\}$. Such functions are called *viewpoints*. By a *confusion* we mean any set of viewpoints. We say that a formula $a$ is *valid in a confusion* $\Upsilon$, when there is $v \in \Upsilon$ such that $v(a) = \top$. And we say that a formula is *valid* when it is valid in all confusions. Moreover we define a consequence relation in the following way: $T \vdash_{\wp\Omega} a$ iff whenever $T$ is valid in a confusion so is $a$. By *a logic of confusion* we mean any consequence relation defined from a set of viewpoints as above.

A logic, i.e. a consequence relation, is said to be *Tarskian* iff it is reflexive, monotonic and transitive. We can then state the following result:

THEOREM 1 *Any logic of confusion is Tarskian.*

To any confusion $\Upsilon$ we can associate a consequence relation defined in the following way: $T \vdash_{\Upsilon} a$ iff whenever $v(T) = \top$, we have $v(a) = \top$. In particular we have the consequence relation $\vdash_{\Omega}$ associated to the set of all viewpoints, the global confusion $\Omega$. We call this, the *logic of global confusion.* Given a logic of confusion we can therefore always associate its logic of global confusion. We have the following result:

THEOREM 2 *A logic of confusion $L2$ is included in a logic of confusion $L1$, when the logic of global confusion of $L2$ is included in the logic of global confusion of $L1$.*

THEOREM 3 *Any logic of confusion is included in its logic of global confusion:*

$$\textit{If } T \vdash_{\wp\Omega} a \textit{ then } T \vdash_{\Omega} a$$

Given a logic we can consider its *thetical logic*, i.e. the set of formulas $a$ such that $\emptyset \vdash a$, and we define straightforwadly its thetical logic of global confusion. We have then:

THEOREM 4 *Given any logic of confusion, its thetical logic is identical to its thetical logic of global confusion:*

$$\emptyset \vdash_{\wp\Omega} a \textit{ iff } \emptyset \vdash_{\Omega} a$$

A binary connective $\star$ is said to be *conjunctive* when for any viewpoint $v$ we have $v(a \star b) = \top$ iff $v(a) = \top$ and $v(b) = \top$. A conjunctive connective $\star$ is said to be *non-adjunctive* when there are formulas $a$ and $b$ such that: $\{a, b\} \nvdash a \star b$. We can now state the following result:

THEOREM 5 *Any conjunctive connective in a logic of confusion is non-adjunctive.*

A *conjunctive logic* is a logic with a conjunctive connective. From THEOREM 3 and THEOREM 5 we have then the following

COROLLARY *Any conjunctive logic of confusion is strictly included in its logic of global confusion.*

In the definition of a logic of confusion viewpoints are *bivaluations*, i.e. functions which take values in a set of two elements. That does not mean that the semantics of a logic of confusion is a bivalent truth-functional semantics. We can take viewpoints which are bivaluations but which are not homomorphisms, like in the case of the bivalent semantics for da Costa's paraconsistent $C_1$ or Suszko's semantics for Lukasiewcz logic $L_3$. We can consider viewpoints which are homomorphisms taking values in a set of any cardinaly divided in two sets: designated values and non-designated values, like in standard many-valued logics. Then we define $v(a) = \bot$ iff $v(a)$ is a non-designated value and $v(a) = \top$ iff $v(a)$ is a designated value. We can also use these definitions when we have viewpoints which are not homomorphisms but functions taking values in a set of any cardinaly divided in two sets of designated values and non-designated values, as suggested in [9].

Let us now examine several examples of logics of confusion.

# 3 Classical logic of confusion

We consider the set of classical bivaluations, that is to say the set of functions from $\mathcal{F}$ into $\{0, 1\}$ defined by the usual bivalent truth-tables. The classical logic of confusion is constructed by taking all these bivaluations as viewpoints identifying 0 with $\bot$ and 1 with $\top$. That means that its logic of global confusion is classical logic.

Due to the results of the preceding section we know that all theses of classical logic are valid in the classical logic of confusion and as classical logic is a conjunctive logic, we know also that the classical logic of confusion is strictly included in classical logic. The question is to know exactly what classical statements are preserved in it.

In the classical logic of confusion all viewpoints are consistent: we cannot have contradictions within a given viewpoint. However a confusion can be inconsistent in the sense that it contains viewpoints that are contradictory and nonetheless this confusion can be non trivial in the sense that not everything is valid in it. To see this it is enough to consider a confusion with two bivaluations, one in which $p$ is true and the other one in which $p$ is false (consequently $\neg p$ is true in it), and such that in both $q$ is false, this shows that: $\{p, \neg p\} \nvdash q$. We have thus:

THEOREM 6 *The classical logic of confusion is paraconsistent.*

Moreover the classical logic of confusion is strictly paraconsistent in the sense that $\{a, \neg a\} \nvdash \neg b$ does not either hold in general, as it can easily be seen.

Due to general results presented in [3], we know that contraposition and *reductio ad absurdum* do not hold in the classical logic of confusion. However we have the following positive result:

THEOREM 7 *The classical logic of confusion is self-extensional.*

The classical logic of confusion is in fact nothing else than Jaśkowski logic, or better: it is a new way to look at it (On this logic see e.g. [14], [18]). It is supported by the idea that a confusion can be considered as a discussion group whose members are rational agents who behave classically. But viewpoints can also be considered as rational agents who behaved non-classically or packages of incomplete and/or inconsistent information, etc.

# 4 Lukasiewicz logic of confusion

Now we construct a logic of confusion taking the set of viewpoints to be Lukasiewicz three-valued valuations. That is to say that viewpoints are defined as follows: $v(a) = \top$ iff $\lambda(a) = 1$ and $v(a) = \bot$ iff $\lambda(a) = 0$ or $\frac{1}{2}$, where $\lambda$ is a three-valued function taking values in Lukasiewicz set of values $\{0, \frac{1}{2}, 1\}$, where only 1 is considered as designated. $L_3$ is therefore the logic of global confusion of Lukasiewicz logic of confusion.

Due to the results of section 2, we know that Lukasiewicz logic of confusion is strictly included in the classical logic of confusion, since $L_3$ is strictly included in classical logic. In particular this logic is a paraconsistent logic.

THEOREM 8 *Lukasiewicz logic of confusion is self-extensional.*

It has some further interesting features from the perspective of paraconsistent logic. Neither the strong law of contradiction $\{a, \neg a\} \vdash b$ nor the weak law of contradiction $\vdash \neg(a \wedge \neg a)$ hold. The strong law does not hold for the same reason as in the classical logic of confusion and the weak law does not hold because it does not hold in $L_3$.

In the case of the classical logic of confusion, and other paraconsistent logics like $LP$ and $J_3$, the strong law does not hold but the weak law holds. In $L_3$ we have an opposite situation since the strong law holds ($L_3$ is not paraconsistent). In both cases there are no intuitive support for the validation of one law and the rejection of the other one. These results are in fact even counter-intuitive whether one interprets the third value as designated (case of $LP$ and $J_3$) or non-designated (case of $L_3$).

Another logic in which both laws of contradiction do not hold is the paraconsistent logic $C_1$ of da Costa. However this logic is not self-extensional. Lukasiewicz logic of confusion can be seen as a possible solution to the so-called *da Costa problem*: finding a logic in which both laws of contradictions do not hold and which is as strong as possible (on other possible solutions to this problem see [1]).

Another interesting feature of Lukasiewicz logic of confusion is that it is *paracomplete* in the sense that the law of excluded middle does not hold. This is due to the fact that this law does not hold in $L_3$. This is interesting for possible applications. For example if we consider that a viewpoint is a physical experiment, this experiment may give no information at all about a given statement, the third-value of Lukasiewicz is in this case used for this lack of information.

The idea to use three-valued logic in physics can be traced back to the work of Paulette Février in the 1930s (see e.g. [16]). It seems to us that combining three-valued logic with Jaskowski-type semantics can lead to interesting results in the field of quantum physics. However maybe Lukasiewicz logic of confusion is not the best solution, due to the fact that it presents some drawbacks at the level of implication.

# 5 Implication and confusion

We say that a binary connective $\star$ is *anti-deductive* when

$$\text{if } T \vdash a \star b \text{ then } T \cup \{a\} \vdash b$$

THEOREM 9 *In a logic of confusion an implication which is not anti-deductive cannot obey the two forms of modus ponens:*

$$\{a, a \rightarrow b\} \vdash b$$

$$\textit{if} \vdash a \textit{ and } \vdash a \to b, \textit{ then} \vdash b$$

An implication $\to$ is said to be *standard* when for any viewpoint $\upsilon$, $\upsilon(a \to b) = \bot$ iff $\upsilon(a) = \top$ and $\upsilon(b) = \bot$.

THEOREM 10 *In a conjunctive logic of confusion a standard implication is not anti-deductive.*

In Lukasiewicz logic of confusion the implication is not standard, but still it is not anti-deductive. This follows from the more particular result below.

THEOREM 11 *If in a logic of confusion we have* $\vdash a \to (b \to a \wedge b)$ *where* $\wedge$ *is conjunctive then this implication is not anti-deductive.*

One may want to improve the situation and to find a logic of confusion which is similar to Lukasiewicz logic, i.e. based on the same kind of three-valuations, but in which the implication is anti-deductive. The idea would be to change the truth-table for implication. Unfortunately no such a change can solve the problem under reasonable conditions. Let us show this.

If we have an anti-deductive implication in a logic of confusion, then we must have: $\{p\} \nvdash q \to (p \wedge q)$ since $\{p, q\} \nvdash p \wedge q$. We must therefore have a confusion $\Upsilon$ such that there is a viewpoint $\upsilon \in \Upsilon$ with $\upsilon(p) = \top$ and such that for every viewpoint $w \in \Upsilon$, $w(q \to (p \wedge q)) = \bot$. If we have viewpoints similar to those of Lukasiewicz logic of confusion, i.e. which are based on functions taking values in a set of three-elements $\{0, \frac{1}{2}, 1\}$ with only the value 1 as designated, then $a \to a$ cannot be a thesis: suppose $\upsilon(p) = \top$, then $\upsilon(p \wedge q) = \upsilon(q)$; since $\upsilon(q \to (p \wedge q)) = \bot$, the implication should be defined in a way such that it is possible for two formulas $a$ and $b$ to have a viewpoint $\upsilon$, such that $\upsilon(a) = \upsilon(b)$ and $\upsilon(a \to b) = \bot$, in this case we will also have $\upsilon(a \to a) = \bot$.

## 6 Conclusion

The technique of logic of confusion permits to develop nice paraconsistent and paracomplete logics which may have some interesting applications. However lots of investigations still have to be done. For example an open problem is to know if it is possible to construct – using two, three or more values, truth-functional or non truth funtional semantics – a logic of confusion in which none of the law of contradiction hold, nor the law of excluded middle and which is conjunctive, self-extensional and anti-deductive.

To solve this problem and some other similar ones, we will use *Universal Logic*, a general theory of logics which provides useful tools for the working logician lost in the contemporary jungle of logics (see [4], [5], [10]).

## References


[1] João Marcos de Almeida. 8K solutions and semi-solutions to a problem of da Costa. Forthcoming.

[2] Mara Beller. The birth of Bohr's complementarity: the context and the dialogues. *Studies in History and Philosophy of Science*, 23:147–180, 1992.

[3] Jean-Yves Béziau. Théorie législative de la négation pure. *Logique et Analyse*, 147-148: 209–225, 1994.

[4] Jean-Yves Béziau. Universal Logic. in *Logica '94 - Proceedings of the 8th International Symposium*, T.Childers and O.Majers (eds), Czech Academy of Science, Prague, 73–93, 1994.

[5] Jean-Yves Béziau. *Recherches sur la logique universelle.* PhD, Department of Mathematics, University of Paris 7, 1995.

[6] Jean-Yves Béziau. Éléments pour un *Tractacus Logico-Physicus.* Research Report, 27, National Laboratory for Scientific Computing, Rio de Janeiro, 1995.

[7] Jean-Yves Béziau. What is many-valued logic? in *27th International Symposium on Multiple-Valued Logic*, IEEE Computer Society, Los Alamitos, 117–121, 1997.

[8] Jean-Yves Béziau. What is paraconsistent logic? in *Frontiers of paraconsistent logic*, D.Batens et al. (eds), Research Studies Press, Baldock, 95–111, 2000.

[9] Jean-Yves Béziau. Non truth-functional many-valuedness. Forthcoming.

[10] Jean-Yves Béziau. Universal logic: Towards a general theory of logics. Bulletin of Symbolic Logic, 7:101, 2001.

[11] David Bohm. *Wholeness and the implicate order.* Routledge, London, 1980.

[12] Arthur Buchsbaum and Tarcisio Pequeno. A family of paraconsistent and paracomplete logics with recursive semantics. Research Report, 14, Institute for Advanced Study, São Paulo, 1993.

[13] Walter Carnielli and Mamede Lima-Marques. Society semantics and multiple-valued logics. in *Advances in comtemporary logic and computer science*, AMS, Providence, 33–52, 1999.

[14] Newton C.A. da Costa and Francisco A. Doria. On Jaśkowski's discussive logics. *Studia Logica*, 54: 33–60, 1995.

[15] Newton C.A. da Costa and Décio Krause. Complementarity and paraconsistency. in *Logic, Epistemology and the unity of science*, Oxford University Press, to appear.

[16] Paulette Destouches Février. Les relations d'incertitude d'Heisenberg et la logique. in *Travaux du IXéme Congrès International de Philosophie, Volume VI*, Hermann, Paris, 88–94, 1937.

[17] Werner Heisenberg. *Physics and philosophy.* Harper, New York, 1958.

[18] Max Urchs. Discursive logic: Towards a logic of rational discourse. *Studia Logica*, 54: 231—249, 1995.

## Acknowledgments

This work was supported by a grant of the *Swiss National Science Foundation.* I would like to thank João Marcos de Almeida, Arthur de Vallauris Buchsbaum, Décio Krause for discussions related to this paper and Patrick Suppes who invited me to work at Stanford University where this work was written.