Abstract
This paper deals with the accurate measurement of a liquid flow for a medical dropper. At the moment, droppers either rely on drop counting for measuring the liquid flow and they are inaccurate or they use precise peristaltic pumps and they are expensive. Because the drops falling from a pipe are subject to volume variations, some form of drop volume measurement is required. To perform this task, a vision system is well suited as it satisfies the requirement of a non-invasive measurement. The paper, thus, analyses main aspects of a vision unit that performs this task and it also presents the developed prototype dropper.

Principle
The basic hypothesis of this work was to consider drops during freefall as solids of revolution. It means that the drops and their contour exhibit a radial symmetry with respect to the vertical axis. The contour is defined as a continuous function Y(X) measuring its distance (on one side) to the vertical axis going through the drop center of mass. As discrete images will be used, the volume will be calculated as a sum of partial cylindrical volumes. To reduce quantization error, both left and right contours of drops (Y_L, Y_R) will be used resulting in the formula (1) of figure 1.

\[ V = \frac{\pi}{4} \sum_{x} \left( Y_R(x) - Y_L(x) \right)^2 \]  
(1)

Setup
Several setups have previously [HUG] been tested and discussed for the drop contour acquisition. The best results according to precision and efficiency were obtained with the setup of figure 2.
Drop detection and counting is performed by means of an infrared light barrier. This triggers, with controlled delay, a flash of diffuse visible backlight. The flash “freezes” the movement of the drop image on a ¼" CMOS sensor. All the system takes place in a compact portable device. This setup is the result of a thorough study of possible artificial vision solutions: coherent light, collimated light, image or shadow of the drop. Some constraints as the versatility of the surrounding plastic cylinder or liquid optical properties led to retain this solution. Transparent drops will act as lenses, imaging the backlight source at their center and its surroundings along their contour (left of fig.3) whereas opaque drops will appear black on a light background. This will help to segment the contour (right of fig. 3) needed for volume calculation. Segmentation is performed more accurately on an image where the background has been subtracted from the drop image (difference image). In order for all the drops commonly used in droppers (17-100μl) to fit on the sensor surface the image enlargement \( \beta \) has been set at 0.5.

Figure 3: Drop, difference and contour images obtained by the prototype

Accuracy
Now that the principle of measure and setup have been defined, the limitations that reduce measurement accuracy will be discussed. Errors might occur due to poor image quality but also because of model inadequacies. Five sources of error were evaluated:

- Discretization error
- Motion blur
- Spatial blur
- Scaling error
- Tilt error
The goal is to have a volume measurement with a precision of 1% or in other words, a signal to noise ratio (SNR) of 40dB. To achieve this, the contribution of each source of error was analyzed to improve the setup.

**Image quality**

The image of the drop on the sensor will be an approximate (discrete, quantized) representation of the real drop due to limitations of the imaging system thus introducing some noise. The importance of such noise was evaluated as a signal to noise ratio of the calculated volume of drops. From statistical error analysis, the volume SNR was calculated as the following expression:

\[
SNR_v = \sqrt{\frac{3}{2} \hat{D}^3}
\]  

(2)

where \(\hat{D} = \frac{D}{e}\) stand for the normalized diameter of the drop against the size of uncertainty \(e\). A cylindrical model of the drop was assumed (it is calculated as a sum of cylindrical slices). Common sense is in good accordance with (2). The longer the diameter of a cylinder is, the less relative influence a given uncertainty \(e\) will have on its volume.

This uncertainty was evaluated for the discretization noise, the motion blur and the spatial blur.

**Discretization noise**

The contour functions \(Y_R, Y_L\) used to calculate the drop volume are discretized to integer values. So, at each point they can vary from the real contour up to a distance corresponding to a half pixel. The associated drop contour position uncertainty is

\[
e = \frac{1}{\beta \cdot R}
\]  

(3)

where \(\beta\) is the enlargement factor (\(=\).5 for the prototype) and \(R\) sensor resolution.

**Motion blur**

The image acquisition requires a lapse of time \(\tau\) to accumulate enough light. So, if during this lapse of time the drop moves, it will result in a blurred image with reduced contrast. This corresponds to a low pass filtering. It could shift the position of detected contour. The extension of motion blur can be calculated. A falling drop has a given velocity \(v\) when it passes in front of the sensor. This velocity has mainly 2 components, a vertical component resulting from earth gravity uniformly affecting the drop and a velocity field resulting from the oscillation initiated at dropout. However the oscillation velocity becomes quickly negligible versus the average speed of the drop. Considering the height of fall \(H\) and earth acceleration \(g\), the blur will have an extension approximately equal to

\[
e = \tau \cdot \sqrt{2 \cdot g \cdot H}
\]  

(4)

**Spatial blur**

If the drop is shifted (\(\Delta Z\)) along the optical axis, it is no longer in the focus plane of the optical system and its image will get blurred. This effect will typically result in a symmetric gaussian blur. The blur extension in the case of an optical system that has an enlargement \(\alpha=0.5\) is given by the formula
\[
e = \sqrt{(24 \cdot k \cdot f^2)^2 + 64 \cdot \Delta Z^2 \cdot f^2 \cdot k^2 - 24 \cdot k \cdot f^2} \over 32 \cdot k^2 \cdot \Delta Z \tag{5}
\]

where \(k\) is the aperture of the lens of focal \(f\).

**Model limitations**

The calculation of the volume on the basis of the drop image relies on two assumptions. First, the ratio between image and drop size (enlargement \(\beta\)) must be constant. Secondly, it is supposed that the drop is symmetric along the vertical axis. We will report, here, the consequences of a violation of these assumptions. The effects of a shift of the drop along the optical axis will be presented, whereas, for the symmetry hypothesis, the case of a tilt of measurement system relatively to vertical will be considered.

**Scaling error**

Many reasons can shift the drop along the optical axis: perfusion misplacement, tilt of the system, horizontal acceleration of the system. The consequence is a wrong calculation of real drop dimensions leading to an error on the volume. At a given enlargement (\(\beta=0.5\) here) this error increases for larger shifts \(\Delta Z\) and decreases with larger focal length \(f\).

\[
SNR = \frac{1}{\left(\frac{3 \cdot f}{3 \cdot f - \Delta Z}\right)^3 - 1} \tag{6}
\]

Note: the size of the drop has no influence on this error (proportional to volume).

**Tilt error**

The influence of a tilt \(\alpha\) of the system regarding to vertical axis was studied. An ellipsoid was taken as an approximation for the drop. For small angles, an upper bound on the error was calculated.

\[
SNR = \frac{1}{\left(\frac{a^2}{b^2 - 1}\right)^{3/2} \sin^2(\alpha)} \tag{7}
\]

\(a\) and \(b\) are the two principal diameters of the ellipse (a\(\geq\)b). Their ratio for the drops is less than 1.5.

To achieve the 40dB goal, equations (2) - (7) translate to precise requirements for the system features. The system has a 12mm focal lens and \(\beta=0.5\) as basis. Sensor resolution must be better than 10 pixel/mm which is achieved even with cheap CIF sensors. Flash duration \(\tau\) must be less than 0.2 ms. Tilt of system should not exceed 5° and drop position uncertainty \(\Delta Z\) must be less than 2mm for the spatial blur and less than 100 \(\mu\)m (!) for the scaling error. This last point is a problem that has been considered.

**Scale change compensation**

The precise measurement of a drop volume by image is only possible if the ratio between the size of drop and the size of its image is known. This ratio will change if the drop is shifted along the optical axis \(Z\) (use of telecentric lens excluded). The cause of the shift can be a measurable tilt of the apparatus, however, other
sources for the shift exist that cannot be neglected. The dropout position might vary, or an acceleration of the system while the drop falls can lead to a shift as well. So, the solution is actually to measure the drop position thanks to a second sensor. This sensor should be orthogonal relatively to the first one. Figure 4 shows this setup. One sensor will have an X-Z view of the drop and the other an X-Y view, completely identifying thus the spatial position of the drop.

![Figure 4: Dual axis measurement setup](image)

**Measurements**

A practical evaluation of the proposed approach to measure drop volume was performed with more than 2000 drops. Drops of variable volumes, falling at different rates with various transparency were, at the same time, measured by the vision system and weighted on a high precision scale placed below the dropper. This was done in order to ensure accuracy but also reliability of the measurement process.

![Figure 5: measurement sample results](image)

Figure 5 displays the measurements of 4 series of 50 drops coming out of 3 different pipes. 60, 20 and 10 drops per ml pipes were used here with transparent water. After calibration ($\beta$ factor calculation), the volume measurements matched the weight given by the scale with a precision of 0.7% for the largest drops and 1.2% for the smallest. This result is satisfying as it is well below the drop volume
variability and around the objective performance. Pipes for droppers have a
typical tolerance of 5-10% on the volume of the drop they create. Furthermore, the
drop volume is dependent on the drop rate. One series of 50 measurements at
center of figure 5 shows the volume variation occurring with variable drop rate
(weight between 48 and 56 mg). The vision system trustworthy detects such
changes as well.

Scale change compensation was also evaluated in the laboratory with a setup
corresponding to fig.4. A pipe generating drops at the same rate with
approximately same weight was shifted along the optical axis of up to 4mm. Drop
position detection allowed to reduce measurement variability from 16 to 0.5%. It
proved its ability to cancel that source of error due to a tilt of the system, prone to
occur in real use of the device.

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Conclusion
The accurate measurement of the drop volume by artificial vision has been
studied in the context of a medical dropper. Some features of the drops as
rotational symmetry or lens behaviour have been used to ease the volume
calculations. A theoretical study of sensitivity to perturbations was done to ensure
the 1% goal accuracy could be achieved. A cheap and compact prototype of the
system has been implemented with power consumption similar to the basic drop
counter.

Reference
[HUG] Hügli H., Gonzalez J., “Drop volume measurements by vision”, SPIE
3966 pp 60-66, San Jose California, January 1999